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VARIETY OF THE REPRESENTATION OF THE NONSTATIONARY TEMPERATURE OF FUEL ELEMENTS DEPENDING ON THE FORM OF THE DISTRIBUTION OF THE HEAT SOURCES

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The author proposes a representation of the solution of the heat-conduction equation in the form of expansions in basis functions that are selected depending on the form of the distribution of the internal heat sources.

The approach to mathematical models of heat conduction with thermal loadings by internal sources that envisions a nonstationary process in continuous media with distributed parameters (temperature, etc.) enables us, on the basis of the general theory of synthesis and analysis of complex systems, to develop them further and offer a new interpretation of results of investigating boundary-value problems of heat and mass transfer by an analytical-numerical method [1]. According to this solving algorithm, solution of the equation

$$\rho(\xi, m)\frac{\partial T}{\partial \mathrm{Fo}} = \frac{\partial}{\partial \xi}\left(\lambda(\xi, m)\frac{\partial T}{\partial \xi}\right) + \frac{q_v(\xi, \mathrm{Fo})\,R^2\xi^m}{\lambda_0} \tag{1}$$

with generalized boundary conditions of the third kind in terms of the Laplace transform $\bar{T}(\xi, p)$ is reduced to the problem

$$\frac{d}{d\xi}\left(\lambda(\xi, m)\frac{d\bar{T}}{d\xi}\right) - \rho(\xi, m)\,[p\bar{T}(\xi, p) - T_0] + \frac{\bar{q}_v(\xi, p)\,R^2\xi^m}{\lambda_0} = 0\,, \tag{2}$$

$$\left\{\frac{d\bar{T}}{d\xi} + \mathrm{Bi}\,\bar{T}(\xi, p)\right\}_{\xi=1} = \mathrm{Bi}\left[\bar{\varphi}(p) + \frac{\bar{q}(p)}{\alpha}\right],\quad \left(\frac{d\bar{T}}{d\xi}\right)_{\xi=0} = 0\,, \tag{3}$$

where $\mathrm{Fo} = at/R^2$; $1 \le \xi = x/R \le 1$ for a plate ($m = 0$); inside a cylinder and a sphere, $0 \le \xi = r/R \le 1$ ($m = 1, 2$); $a = \lambda_0/c\gamma$ is the conventional thermal diffusivity, which is equal to the actual one with constant thermophysical coefficients; the function $\Phi(\mathrm{Fo}) = \varphi(\mathrm{Fo}) + q(\mathrm{Fo})/\alpha$ is the generalized temperature of the external medium; $q(\mathrm{Fo})$ is the radiational influx ($q > 0$) or radiation ($q < 0$) of heat.

The kernel $\bar{R}(\xi, \mu)$ of the integral transform of the expression $\frac{\partial}{\partial \xi}\left(\lambda(\xi, m)\frac{\partial T}{\partial \xi}\right)$ and the system of eigenfunctions of Eq. (1) are found from the soution of the equation

$$\frac{d}{d\xi}\left(\lambda(\xi, m)\frac{d\bar{R}}{d\xi}\right) + \mu^2\rho(\xi, m)\,\bar{R}(\xi, m) = 0\,. \tag{4}$$

For $\lambda(\xi, m) = \xi^m$ and $\rho(\xi, m) = \xi^m$, the solutions of this equation are trigonometric ($m = 0, 2$) and Bessel ($m = 1$) functions. Such a system of eigenfunctions of the corresponding Sturm–Liouville problem forms the basis of a strict functional space. Synthesis of the sought solution requires that all input values of the

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thermal loadings in this space be analyzed in order to represent the temperature in the form of an infinite series, which most often converges poorly, especially at small Fo numbers. Therefore A. V. Luikov, in the theory of heat conduction, proposed another solution that makes it possible to perform an efficient thermal calculation for the initial period of heating [2].

The variety of the representation of solutions in different alternative spaces whose bases are selected depending on the form of the stationary distribution of the internal heat sources makes it possible to find the temperature fields in the best approximations. We seek the solution of the boundary-value problem (2) and (3) for $\lambda(\xi, m) = \xi^m$, $\rho = \xi^m$, and $q_v(\xi, \mathrm{Fo}) = q_v\varphi_0(\xi)f(\mathrm{Fo})$ in the space

$$\left\{\psi_1(\xi), \psi_2(\xi), ..., \psi_n(\xi)\right\} \tag{5}$$

in the form

$$\bar{T}_n(\xi, p) = \bar{\Phi}(p) + \sum_{k=1}^{n} \bar{a}_k(p)\,\psi_k(\xi), \tag{6}$$

where the alternative of selecting the coordinate functions ψ_k is confined just to homogeneous boundary conditions (3):

$$\left(\frac{d\psi_k}{d\xi}\right)_{\xi=0} = 0, \quad \left\{\frac{d\psi_k}{d\xi} + \mathrm{Bi}\,\psi_k(\xi)\right\}_{\xi=1} = 0, \quad \forall\, k = 1, 2, ..., n. \tag{7}$$

The procedure for realization of the orthogonal projection of the discrepancy leads to the matrix transformation

$$\|A + pB\|\,\|\bar{a}(p)\| = [T_0 - p\,\bar{\Phi}(p)]\,\|C\| + \frac{q_v R^2}{\lambda}\bar{f}(p)\,\|D\|, \tag{8}$$

where the matrix elements are found from the formulas

$$A_{jk} = -\int_0^1 \frac{d}{d\xi}\left(\xi^m \frac{d\psi_k}{d\xi}\right)\psi_j(\xi)\,d\xi = \int_0^1 \psi_k'\,\psi_j'\,\xi^m\,d\xi + \mathrm{Bi}\,(\psi_k\psi_j)_{\xi=1} = A_{kj} > 0,$$

$$B_{jk} = \int_0^1 \psi_j\,\psi_k\,\xi^m\,d\xi = B_{kj} > 0, \quad C_j = \int_0^1 \psi_j\,\xi^m d\xi, \quad D_j = \int_0^1 \varphi_0(\xi)\;\psi_j\,\xi^m d\xi. \tag{9}$$

The solution of system (8) by the Cramer formula will be

$$\bar{a}_k(p) = \frac{[T_0 - p\bar{\Phi}(p)]\,\Delta_k^{(C)}(p)}{\Delta(p)} + \frac{q_v R^2}{\lambda}\frac{\bar{f}(p)\,\Delta_k^{(D)}(p)}{\Delta(p)}, \tag{10}$$

where $\Delta_k^{(N)}(p) = \sum_{j=1}^{n} N_j\Delta_{jk}(p)$; $\Delta_{jk}(p)$ are the algebraic complements of the basic determinant $\Delta(p) = |A + pB|$.

Upon passing from problem (2) and (3) to transformation (8) the matrices $\|A\|$ and $\|B\|$ interpolate the self-adjoint differential operator along the elliptic coordinate ξ in Eq. (1); therefore, as formulas (9) confirm, they are symmetric and positive.

Consequently, the roots of the equation $\Delta(p) = 0$ will be different and negative. We denote them by $p_k = -p_k^{(n)}$ in the ascending order of positive numbers $p_1^{(n)} < p_2^{(n)} < ... < p_n^{(n)}$ $(p_k^{(n)} > 0)$.

The transfer functions $\Delta^{(N)}(p)/\Delta(p)$ are proper fractions in structure of representation and by expanding them in simple poles of the denominator in (10) we obtain

$$\bar{a}_k(p) = \sum_{i=1}^{n} \frac{\Delta_k^{(C)}(-p_i^{(n)})}{\Delta'(-p_i^{(n)})} \frac{[T_0 - p\bar{\Phi}(p)]}{p + p_i^{(n)}} + \frac{q_v R^2}{\lambda} \sum_{i=1}^{n} \frac{\Delta_k^{(D)}(-p_i^{(n)})}{\Delta'(-p_i^{(n)})} \frac{\bar{f}(p)}{p + p_i^{(n)}} \tag{11}$$

which is the formula of synthesis of the elements of the matrix-response $\|\bar{a}(p)\|$ to sums of blocks of elementary inertial links. This representation makes it possible, for specific forms of the thermal loadings $\Phi(p)$ and $f(p)$, to take inverse Laplace transforms of the same type in each block and write the temperature (6) in the domain of the inverse transforms.

According to the method of selecting basis coordinates [1] for q_v (ξ, Fo) = $q_v f$(Fo) and $\varphi_0(\xi) = 1$ the optimum system will be $\psi_k(\xi) = \frac{\mathrm{Bi} + 2k}{\mathrm{Bi}} - \xi^{2k}$, and the coefficients (9) are easily written in terms of recurrence formulas in the integers i, k, and m, which enables us to compose a program for writing the algebraic system (8) in explicit form of any order for each body individually (m = 0, 1, 2) and for $n \geq 3$ for a specific Bi number.

From the truncated system of first order,

$$\bar{a}_1(p) = \frac{A(\mathrm{Bi}, m)\,[T_0 - p\bar{\Phi}(p)]}{2(m+1)\,[p + A(\mathrm{Bi}, m)]} + \frac{q_v R^2}{2\lambda(m+1)} \frac{\bar{f}(p)}{p + A(\mathrm{Bi}, m)}, \tag{12}$$

where

$$A(\mathrm{Bi}, m) = \frac{\mathrm{Bi}\,(m+1)(m+5)\,[\mathrm{Bi} + (m+3)]}{2\mathrm{Bi}^2 + 2(m+5)\,\mathrm{Bi} + (m^2 + 8m + 15)}. \tag{13}$$

For $\Phi(\mathrm{Fo}) = T_{\mathrm{m}} + q/\alpha$ = const and $f(\mathrm{Fo}) = 1$, the relative excess temperature in fuel elements of the three geometric shapes is found by the single formula

$$\theta(\xi, \mathrm{Fo}, \mathrm{Bi}, m) = \frac{T(\xi, \mathrm{Fo}) - T_0}{(T_{\mathrm{m}} + q/\alpha) - T_0} =$$

$$= 1 - \frac{A(\mathrm{Bi}, m)}{2(m+1)} \left(\frac{\mathrm{Bi} + 2}{\mathrm{Bi}} - \xi^2 \right) \exp[-A(\mathrm{Bi}, m)\,\mathrm{Fo}] +$$

$$+ \frac{q_v R^2}{2\lambda(m+1)[(T_{\mathrm{m}} + q/\alpha) - T_0]} \left(\frac{\mathrm{Bi} + 2}{\mathrm{Bi}} - \xi^2 \right) \{1 - \exp[-A(\mathrm{Bi}, m)\,\mathrm{Fo}]\}, \tag{14}$$

where T_{m} is the temperature of the medium.

The temperature changes due only to the internal sources for f(Fo) = 1 − exp (− Pd Fo) and f(Fo) = 1 + Fo exp (−Pd Fo) are determined by the expressions

$$T(\xi, \mathrm{Fo}, \mathrm{Bi}, \mathrm{Pd}, m) = T_0 + \frac{q_v R^2}{2\lambda(m+1)} \left(\frac{\mathrm{Bi} + 2}{\mathrm{Bi}} - \xi^2 \right) \times$$

$$\times \left\{ 1 - \frac{\mathrm{Pd} \exp[-A(\mathrm{Bi}, m)\,\mathrm{Fo}] - A \exp(-\mathrm{Pd}\,\mathrm{Fo})}{\mathrm{Pd} - A(\mathrm{Bi}, m)} \right\}, \tag{15}$$

TABLE 1. Coefficients A and First Eigenvalues of the Characteristic Equations μ_1 [1] for Different Bi

m	$A(\mathrm{Bi}, m), \mu_1^2$	Bi									
		0.	0.02	0.1	0.4	0.8	1.0	2.0	10	50	∞
0	$A(\mathrm{Bi}, 0)$	0.0	0.02	0.097	0.352	0.626	0.741	1.163	2.063	2.063	2.500
	$\mu_1^2(0)$	0.0	0.02	0.097	0.352	0.626	0.740	1.160	2.042	2.042	2.467
1	$A(\mathrm{Bi}, 1)$	0.0	0.141	0.195	0.725	0.321	1.579	2.571	4.884	5.761	6.00
	$\mu_1^2(1)$	0.0	0.141	0.195	0.725	0.320	1.577	2.558	4.750	5.556	5.783
2	$A(\mathrm{Bi}, 2)$	0.0	0.060	0.294	1.109	2.052	2.471	4.141	8.400	10.069	10.50
	$\mu_1^2(2)$	0.0	0.060	0.294	1.108	2.051	2.467	4.116	8.045	9.486	9.870

$$T(\xi, \mathrm{Fo}, \mathrm{Bi}, \mathrm{Pd}, m) = T_0 + \frac{q_v R^2}{2(m+1)\lambda}\left(\frac{\mathrm{Bi}+2}{\mathrm{Bi}} - \xi^2\right) \times$$

$$\times \left\{1 - \exp(-A\,\mathrm{Fo}) + \frac{A}{A-\mathrm{Pd}}\left[\frac{\exp(-A\,\mathrm{Fo}) - \exp(-\mathrm{Pd}\,\mathrm{Fo})}{A-\mathrm{Pd}} + \right.\right.$$

$$\left.\left. + \mathrm{Fo}\exp(-\mathrm{Pd}\,\mathrm{Fo})\right]\right\}, \tag{16}$$

which satisfy all the boundary conditions of the problem and, after the transient regime, coincide with the exact solution

$$\lim_{\mathrm{Fo}\to\infty} T(\xi, \mathrm{Fo}, \mathrm{Bi}, \mathrm{Pd}, m) = T_0 + \frac{q_v R^2}{2\lambda(m+1)}\left(\frac{\mathrm{Bi}+2}{\mathrm{Bi}} - \xi^2\right). \tag{17}$$

In solutions in the second and subsequent approximations for the class of problems with the conditions

$$\lim_{\mathrm{Fo}\to\infty} f(\mathrm{Fo}) = \lim_{p\to 0} p\bar{f}(p) = 1$$

we will have the limiting equalities

$$\lim_{\mathrm{Fo}\to\infty} a_1(\mathrm{Fo}) = \lim_{p\to 0} p\bar{a}_1(p) = \frac{q_v R^2}{2\lambda(m+1)},$$
$$\lim_{\mathrm{Fo}\to\infty} a_k(\mathrm{Fo}) = \lim_{p\to 0} p\bar{a}_k(p) = 0, \quad \forall k \ge 2, \tag{18}$$

i.e., the property (17) is retained and the temperature is refined only on the small time interval of the transient regime. The indicators of the rate of stabilization in formulas (14)-(16) should be comparable with the first eigenvalues of the characteristic equations [2]. Such comparisons are given in Table 1.

The approximate solution (14) virtually coincides with the exact one for the numbers $0 < \mathrm{Bi} \le 1$ for all values of Fo, while for the remaining Bi there is satisfactory agreement for the period $\mathrm{Fo} \ge 0.05$.

For the temperature inside a round bar with constant heat sources, from a partial sum and even the infinite series of the exact solution

$$\theta(\xi, \mathrm{Fo}, \mathrm{Bi}) = \frac{T(\xi, \mathrm{Fo}) - T_0}{(T_\mathrm{m} + q/\alpha) - T_0} = \sum_{k=1}^{\infty} A_k J_0(\mu_k \xi)\,[1 - \exp(-\mu_k^2 \mathrm{Fo})] +$$

$$+ \mathrm{Po}\sum_{k=1}^{\infty} \frac{A_k J_0(\mu_k \xi)}{\mu_k^2}[1 - \exp(-\mu_k^2 \mathrm{Fo})], \tag{19}$$

TABLE 2. Expressions for the Coordinate Functions $\psi_k^{(n)}$ and Roots of the Equation $\Delta(p) = 0$ for $\Phi(\mathrm{Fo}) = T_0$ and $f(\mathrm{Fo}) = 1$ for Different n and k

n	k	$\psi_k^{(n)}(\xi)$	$p_k^{(n)}$
1	1	$1-\xi^2$	6
2	1	$1.1000 - 1.5288\xi^2 + 0.4288\xi^4$	5.7841
	2	$-0.1000 + 0.5288\xi^2 - 0.4288\xi^4$	36.882
3	1	$1.1076 - 1.5984\xi^2 + 0.56603\xi^4 - 0.0752\xi^6$	5.7832
	2	$-0.1308 + 0.8900\xi^2 - 1.2800\xi^4 + 0.5208\xi^6$	30.712
	3	$0.0232 - 0.2916\xi^2 + 0.7140\xi^4 - 0.4456\xi^6$	113.50

where

$$A_k = \frac{2J_1(\mu_k)}{\mu_k\,[J_0^2(\mu_k) + J_1^2(\mu_k)]}\,; \quad \mathrm{Po} = \frac{q_v R^2}{\lambda\,[(T_\mathrm{m} + q/\alpha) - T_0]}\,,$$

we can no longer find explicitly the limiting property (17). Only upon taking into account that with the aim of synthesizing the sought quantity in the form of (19) the input thermal loadings (of the external source $T_\mathrm{m} + q/\alpha \neq T_0$ and the internal source $q_v(\xi, \mathrm{Fo}) = q_v = \mathrm{const}$) were analyzed and another unknown quantity $0.25\left(\frac{\mathrm{Bi}+2}{\mathrm{Bi}} - \xi^2\right)$ that is related to the solution of the stationary problem was additionally synthesized:

$$1 = \sum_{k=1}^{\infty} A_k J_0(\mu_k\xi)\,, \quad \sum_{k=1}^{\infty} \frac{A_k J_0(\mu_k\xi)}{\mu_k^2} = \frac{1}{4}\left(\frac{\mathrm{Bi}+2}{\mathrm{Bi}} - \xi^2\right),$$

can we, by introducing these values into (19), improve the convergence of the series and obtain the limiting equality (17).

In the solving algorithm, the greatest computational error is obtained when $\mathrm{Bi} \to \infty$, and for the cylinder the refinement of the solution in subsequent approximations with boundary conditions of the first kind in a space with the coordinate functions $\psi_k(\xi) = 1 - \xi^{2k}$ for $\Phi(\mathrm{Fo}) = T_0$ and $f(\mathrm{Fo}) = 1$ leads to the expression

$$\theta_n(\xi, \mathrm{Fo}) = \frac{4\lambda\,[T_n(\xi, \mathrm{Fo}) - T_0]}{q_v R^2} = 1 - \xi^2 - \sum_{k=2}^{n} \psi_k^{(n)}(\xi) \exp\left(-p_k^{(n)}\mathrm{Fo}\right), \tag{20}$$

calculation results are given in Table 2.

A considerable excess of $p_n^{(n)}$ over the exact value μ_n^2 and a large deviation of $\psi_n^{(n)}$ from the exact eigenfunction follow from the fact that expression (20) interpolates the polynomial of exponential functions [3] by a smaller number of components than in the partial sum of the exact solution. For example, the solution (20) for $n = 3$ is equivalent to the partial sum of fifth order in convergence.

Temperature profiles at various instants and a comparison of the third approximation with exact values are given in Fig. 1. The temperature $\theta_2(\xi, \mathrm{Fo})$ virtually coincides with $\theta_3(\xi, \mathrm{Fo})$ and gives only very small disagreements in the time interval $0.01 \le \mathrm{Fo} \le 0.05$.

For the parabolic distribution $q_v(\xi, \mathrm{Fo}) = q_v f(\mathrm{Fo})(1 + \delta\xi^2)$, the optimum basis coordinate will be $\psi_1 = \frac{2(m+3)(\mathrm{Bi}+2) + \delta(m+1)(\mathrm{Bi}+4)}{\mathrm{Bi}} - 2(m+3)\xi^2 - \delta(m+1)\xi^4$, and the variety of the representation

$$\bar{T}_n(\xi, p, \mathrm{Bi}, m) = \bar{\Phi}(p) + \bar{a}_1(p)\,\psi_1(\xi) + \sum_{k=2}^{n} \bar{a}_k(p)\left(\frac{\mathrm{Bi}+2(k+1)}{\mathrm{Bi}} - \xi^{2(k+1)}\right) \tag{21}$$

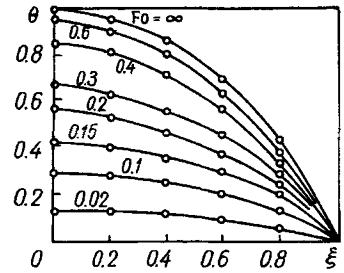

Fig. 1. Temperature distribution along the radius in a fuel element of circular cross section: solid lines) exact solution; points) calculation by formula (20) for $n = 3$.

for a series of problems with the conditions $\lim_{p \to 0} p\bar{f}(p) = \lim_{\text{Fo} \to \infty} f(\text{Fo}) = 1$ leads to the equalities

$$\lim_{p \to 0} p\bar{a}_1(p) = \lim_{\text{Fo} \to \infty} a_1(\text{Fo}) = \frac{q_v R^2}{4\lambda (m+1)(m+3)}, \lim_{p \to 0} p\bar{a}_k(p) = 0, \quad \forall k \geq 2, \tag{22}$$

i.e., the solution (21) for $p \to 0$ (Fo $\to \infty$) will coincide with the exact temperature of the stationary problem.

From the truncated system (8) of first order for $m = 1$ and $\Phi(\text{Fo}) = T_0$,

$$\bar{a}_1(p) = \frac{q_v R^2}{16\lambda} \frac{A(\text{Bi}, \delta) \bar{f}(p)}{p + A(\text{Bi}, \delta)}, \tag{23}$$

$$A(\text{Bi}, \delta) = \frac{10\text{Bi}\,[\text{Bi}\,(3\delta^2 + 16\delta + 24) + 24\,(\delta^2 + 4\delta + 4)]}{\text{Bi}^2 (4\delta^2 + 25\delta + 40) + 40\text{Bi}\,(\delta^2 + 5\delta + 6) + 120\,(\delta + 2)^2}. \tag{24}$$

For a stationary heat source ($f(\text{Fo}) = 1$),

$$T(\xi, \text{Fo}, \text{Bi}, \delta) = T_0 + \frac{q_v R^2}{16\lambda} \left(\frac{\text{Bi}\,(4 + \delta) + (8 + 4\delta)}{\text{Bi}} - 4\xi^2 - \delta\xi^4 \right) \times$$

$$\times \{1 - \exp\,[-A(\delta, \text{Bi})\,\text{Fo}]\}, \tag{25}$$

whence for $q_v(\xi, \text{Fo}) = q_v(1 - \xi^2)$ we find

$$T(\xi, \text{Fo}, \text{Bi}, -1) = \frac{q_v R^2}{16\lambda} \left(\frac{3\text{Bi} + 4}{\text{Bi}} - 4\xi^2 + \xi^4 \right) \{1 - \exp\,[-A(\text{Bi}, -1)\,\text{Fo}]\}. \tag{26}$$

The quantity $A(\text{Bi}, -1)$ by formula (24) better agrees with $\mu_1^2(\text{Bi})$ than expression (13) at $m = 1$. However a uniform excess that exists in formula (13) with increase in the Bi number is already broken for $A(\text{Bi}, -1)$.

According to formula (24) the quantity $A(\text{Bi}, -1)$ agrees better with $\mu_1^2(\text{Bi})$ than expression (13) for $m = 1$. However the uniform excess that exists in formula (13) with increase in the Bi number is already broken for $A(\text{Bi}, -1)$.

According to (24) $A(\infty, -1) = 5.7895$, and it exceeds the exact value $\mu_1^2(\infty) = 5.7831$ by only 0.1%, while at the point Bi = 1 we have a maximum deviation of 1.34%. Unlike the process of heat conduction, for $q_v(\xi, \text{Fo}) = q_v = \text{const}$ the monotonicity of the deviation from the exact value $\mu_1^2(\text{Bi})$ is broken when the character of the dependence of the temperature stabilization on the Bi number is somewhat different because of the nonuniformity of the distribution of the local heat sources. For example, in another parabolic distribution $q_v(\xi, \text{Fo}) = q_v(1 + \xi^2)f(\text{Fo})$, according to formula (24) we have $A(10; 1) = 5.055$, $A(\infty, 1) = 6.232$, and the considerable excesses over the exact eigenvalues $\mu_1^2(10) = 4.750$ and $\mu_1^2(\infty) = 5.783$ are validated by the fact that the largest release of heat occurs in the layer at the body surface $\xi = 1$ and this heat is removed more rapidly to the external medium, which governs the high coefficients of the rate of stabilization of the temperatures. We note that the rate of exponential stabilizations in synthesis spectra of the fuel-element temperature in a strict nonalternative space does not depend on the character of the distribution of the heat sources, i.e., the principle of optimum representation of the solution is absent.

By virtue of the nonuniformity of the neutron power flux the distributions of the heat sources in the fuel elements of the cores of nuclear power plants become coordinate- and time-variable. In [4], B. S. Petukhov et al. propose interpolation of the stationary part of such a distribution by trigonometric or Bessel functions.

For a plate ($0 \le \xi \le 1$) with a source $q_v(\xi, \mathrm{Fo}) = q_v \sin \pi\xi f(\mathrm{Fo})$ with the boundary conditions $T(0, \mathrm{Fo}) = T(1, \mathrm{Fo}) = T_0$ the solution (6) in the space $\{\psi_k(\xi) = \sin \pi k\xi\}$ leads to

$$\bar{a}_1(p) = \frac{q_v R^2}{\lambda} \frac{\bar{f}(p)}{p + \pi^2}, \quad \bar{a}_k(p) = 0, \quad \forall\, k \neq 1, \tag{27}$$

and for a linear rise of the heat sources ($f(\mathrm{Fo}) = \mathrm{Fo}$) we find the exact solution

$$T(\xi, \mathrm{Fo}) = T_0 + \frac{q_v R^2}{\lambda \pi^2} \sin \pi\xi \left\{ \mathrm{Fo} - \frac{1 - \exp(-\pi^2 \mathrm{Fo})}{\pi^2} \right\}. \tag{28}$$

Inside a round bar with $q_v(\xi, \mathrm{Fo}) = q_v J_0(\mu_i \xi) f(\mathrm{Fo})$, where μ_i is a root of the equation $J_0(\mu)/J_1(\mu) = \mu/\mathrm{Bi}$, the representation (6) for $\{\psi_k = J_0(\mu_k \xi)\}$ leads to the formulas

$$\bar{a}_i(p) = \frac{q_v R^2}{\lambda} \frac{\bar{f}(p)}{p + \mu_i^2}, \quad \bar{a}_k(p) = 0, \quad \forall\, k \neq i. \tag{29}$$

The exact solutions for an arbitrary $f(\mathrm{Fo})$, $f(\mathrm{Fo}) = \mathrm{Fo}^2$, and $f(\mathrm{Fo}) = \exp(\delta \mathrm{Fo})$ are equal to

$$T(\xi, \mathrm{Fo}) = T_0 + \frac{q_v R^2}{\lambda} J_0(\mu_i \xi) \left\{ \int_0^{\mathrm{Fo}} f(\tau) \exp[-\mu_i^2 (\mathrm{Fo} - \tau)]\, d\tau \right\}, \tag{30}$$

$$T(\xi, \mathrm{Fo}) = T_0 + \frac{q_v R^2}{\lambda} J_0(\mu_i \xi) \left\{ \mu_i^{-6} [\mu_i^4 \mathrm{Fo}^2 - 2\mathrm{Fo}\, \mu_i^2 + 2\ (1 - \exp(-\mu_i^2 \mathrm{Fo}))] \right\}, \tag{31}$$

$$T(\xi, \mathrm{Fo}) = T_0 + \frac{q_v R^2}{\lambda} J_0(\mu_i \xi) \left\{ \frac{\exp(\delta\, \mathrm{Fo}) - \exp(-\mu_i^2 \mathrm{Fo})}{\delta + \mu_i^2} \right\}. \tag{32}$$

In the previous problem inside a plate we consider another positive source $q_v(\sin \pi\xi + 0.5 \sin 2\pi\xi) f(\mathrm{Fo})$, and then determination of the solution (6) for the first principal (optimum) coordinate function $\psi_1 = 8 \sin \pi\xi + \sin 2\pi\xi$ yields

$$\bar{a}_1(p) = \frac{q_v R^2}{8\pi^2 \lambda} \frac{p_1 \bar{f}(p)}{p + p_1}, \quad p_1 = \frac{68}{65} \pi^2 = 10.325, \tag{33}$$

whence for $f(\mathrm{Fo}) = 1$

$$T(\xi, \mathrm{Fo}) = T_0 + \frac{q_v R^2}{\lambda}\left(\frac{\sin \pi\xi}{\pi^2} + \frac{\sin 2\pi\xi}{8\pi^2}\right)[1 - \exp(-10.325\,\mathrm{Fo})] . \tag{34}$$

The solution of this problem in the strict space $\{\psi_k(\xi) = \sin \pi k\xi\}$ leads to the coefficients

$$\bar{a}_1(p) = \frac{q_v R^2}{\lambda}\frac{\bar{f}(p)}{p+\pi^2}, \quad \bar{a}_2(p) = \frac{q_v R^2}{2\lambda}\frac{\bar{f}(p)}{p+4\pi^2}, \quad \bar{a}_k(p) = 0, \quad \forall k \geq 3, \tag{35}$$

and instead of (34) we obtain the exact solution

$$T(\xi, \mathrm{Fo}) = T_0 + \frac{q_v R^2}{\lambda\pi^2}\sin \pi\xi \left\{1 - \exp(-\pi^2 \mathrm{Fo})\right\} + \frac{q_v R^2}{8\lambda\pi^2} \times$$

$$\times \sin 2\pi\xi \left\{1 - \exp(-4\pi^2 \mathrm{Fo})\right\} . \tag{36}$$

Thus, in determining the solution (19) we expanded the source $q_v(\xi, \mathrm{Fo}) = q_v = \mathrm{const}$ into an infinite number of internal sources with nonuniform distributions, and under the sign of the second sum we have the results of a search for the eigensolutions of these sources. At the same time, a direct search, by a constant internal source, for its own response in the space of power polynomials led to the representation of the temperature by the simple formula (14) along the optimum axis $\psi_1(\xi) = \frac{\mathrm{Bi}+2}{\mathrm{Bi}} - \xi^2$ in the best approximation. Clearly the same calculations were performed in determining the solution (34), which with an indicator of the rate of stabilization of 10.325 ($\pi^2 < 10.325 < 4\pi^2$), rapidly coincides with the exact solution.

For the temperature field $T(\xi, \mathrm{Fo})$ inside a round bar of finite length ($0 \leq \xi = r/R \leq 1$; $0 \leq \eta = z/h \leq 1$) with heat insulation of the ends $z = 0$ and $z = h$ and heat removal through the peripheral surface to a medium with a temperature T_0 with the source $q_v(\xi, \eta, \mathrm{Fo}) = q_v J_0(\mu_i\xi)(1 + \cos \pi\eta) f(\mathrm{Fo})$ the principal coordinate function will be

$$\psi_1(\xi, \eta) = \left(\frac{1}{\mu_i^2} + \frac{\cos \pi\eta}{\mu_i^2 + \pi^2\beta^2}\right) J_0(\mu_i\xi), \quad \beta = \frac{R}{h}. \tag{37}$$

The solution in the form (6) of the best approximation leads to the coefficient

$$\bar{a}_1(p) = \frac{q_v R^2}{\lambda}\frac{A(\mu_i, \beta)\bar{f}(p)}{p + A(\mu_i, \beta)}, \quad A(\mu_i, \beta) = \frac{\mu_i^2 (3\mu_i^2 + 2\pi^2\beta^2)(\mu_i^2 + \pi^2\beta^2)}{\mu_i^4 + 2(\mu_i^2 + \pi^2\beta^2)^2}, \tag{38}$$

and the temperature for $f(\mathrm{Fo}) = 1$ is equal to

$$T(\xi, \eta, \mathrm{Fo}) = T_0 + \frac{q_v R^2}{\lambda}\left(\frac{1}{\mu_i^2} + \frac{\cos \pi\eta}{\mu_i^2 + \pi^2\beta^2}\right) J_0(\mu_i\xi)\left\{1 - \exp[-A(\mu_i, \beta)\,\mathrm{Fo}]\right\} . \tag{39}$$

If we consider two axes with the coordinate functions $\psi_1(\xi, \eta) = J_0(\mu_1\xi)$ and $\psi_2(\xi, \eta) = J_0(\mu_i\xi) \cos \pi\eta$, we obtain

$$\bar{a}_1(p) = \frac{q_v R^2}{\lambda}\frac{\bar{f}(p)}{p + \mu_i^2}, \quad \bar{a}_2(p) = \frac{q_v R^2}{\lambda}\frac{\bar{f}(p)}{p + (\mu_i^2 + \pi^2\beta^2)} \tag{40}$$

and instead of (39) we find the exact temperature from the two eigenspectra of the expansion

$$T(\xi, \eta, \mathrm{Fo}) = \frac{q_v R^2}{\lambda \mu_i^2} J_0(\mu_i \xi) \left\{1 - \exp(-\mu_i^2 \mathrm{Fo}\right\} +$$

$$+ \frac{q_v R^2 \cos \pi\eta}{\lambda(\mu_i^2 + \pi^2\beta^2)} J_0(\mu_i \xi) \left\{1 - \exp[-(\mu_i^2 + \pi^2\beta^2)\, \mathrm{Fo}]\right\}. \tag{41}$$

In all the found one-component representations of the temperatures, we denote the expressions in the braces { ... } by $Q(\mathrm{Fo})$ and the function of the running coordinates ξ and η by $\theta(\xi)$ or $\theta(\xi, \eta)$, and then we obtain

$$\frac{\lambda [T(\xi, \eta, \mathrm{Fo}) - T_0]}{q_v R^2\, Q(\mathrm{Fo})} = \theta(\xi, \eta). \tag{42}$$

For example, for the temperature (15) we have

$$Q(\mathrm{Fo}) = \left\{1 - \frac{\mathrm{Pd} \exp[-A(\mathrm{Bi}; m)\, \mathrm{Fo}] - A \exp(-\mathrm{Pd\, Fo})}{\mathrm{Pd} - A}\right\},$$

$$\theta(\xi) = \frac{1}{m+1}\left(\frac{\mathrm{Bi} + 2}{\mathrm{Bi}} - \xi^2\right).$$

According to [5], the family $\theta(\xi, \eta) = \theta_i = \mathrm{const}$ will be called the set of geometric images (generalized isothermal surfaces) on which the fuel-element temperature is similar to the isothermal surfaces of stationary $\left(\lim\limits_{\mathrm{Fo} \to \infty} f(\mathrm{Fo})\right) = 1)$ or quasistationary $\left(\lim\limits_{\mathrm{Fo} \to \infty} f(\mathrm{Fo})\right) \neq$ const) regimes at any instant. Consequently, inside the heat-releasing body (the fuel element), depending on the thermal state (on the form of the distribution of the heat sources) we determined the variety of Riemannian spaces in which the system of isothermal surfaces that is prescribed at a certain instant remains a system of isothermal surfaces at any instant. The necessity of solving this problem in a more general formulation was stated as early as 1861 by the Paris Academy of Sciences [5], and a fundamental theoretical investigation was performed by B. Riemann.

The solving algorithm makes it possible to conduct similar investigations in heat-releasing bars with a two-dimensional profile of the cross section (a triangle, a sector of a circle, an ellipse, a segment of a parabola, etc.).

For a rectangular profile of the cross section $D\{-h \le x \le h, -b \le y \le b\}$, the heat-conduction equation in the variables $\xi = x/h$, $\eta = y/b$, and $\mathrm{Fo} = at/h^2$ for the transform $\bar{T}(\xi, \eta, p)$ is reduced to the form

$$\frac{\partial^2 \bar{T}}{\partial \xi^2} + \beta^2 \frac{\partial^2 \bar{T}}{\partial \eta^2} - [p\bar{T}(\xi, \eta, p) - T_0] + \frac{q_v(\xi, \eta)\, h^2}{\lambda} \bar{f}(p) = 0, \quad \beta = \frac{h}{b}. \tag{43}$$

In induction heating of a metal bar of square cross section ($\beta = 1$), the internal source is found [6] using the function

$$q_v(\xi, \eta) f(\mathrm{Fo}) = \frac{15 q_v h^2}{16\lambda} [\xi^2 (1 - \eta^4) + \eta^2 (1 - \xi^4)] f(\mathrm{Fo}). \tag{44}$$

The problem of determining the temperature field for $f(\mathrm{Fo}) = 1$ and heat insulation of all four sides is solved in [1], where the isothermal surfaces and the lines of distribution of equal strengths of the heat sources (44) are given. It is significant that the temperature fields inside all bars with the above cross-sectional profiles with boundary conditions of the first kind are determined within the linear composition

$$\bar{T}_n(\xi, \eta, p) = \frac{T_0}{p} + \sum_{k=1}^{n} \bar{a}_k(p)\, \psi_k(\xi, \eta) \tag{45}$$

as the solution of the same equation (43) but in different functional spaces $\{\psi_k(\xi, \eta)\}$ whose coordinate functions are related to composite functions of the boundaries of the regions D. For example, for a rectangular tetragon the composite equation of the boundary of the region D will be $(1-\xi^2)(1-\eta^2) = 0$ and the composite function $\omega(\xi, \eta) = (1-\xi^2)(1-\eta^2) \geq 0$, $\forall \xi, \eta \in D$, while for an isosceles triangle $D\{y \leq \frac{b}{h}x,\ y \geq -\frac{b}{h}x,\ 0 \leq x \leq h\}$ we will have $\omega(\xi, \eta) = (\xi^2 - \eta^2)(1-\xi)$. Inside a parabolic segment $D\{x \geq \frac{h}{b_2}y^2,\ 0 \leq x \leq h\}$ the composite boundary function is equal to $\omega(\xi, \eta) = (\xi - \eta^2)(1-\xi)$; it vanishes on the boundary, and $\omega > 0$ inside D.

For the heat source $q_v(\xi, \eta) f(\mathrm{Fo}) = q_v(2 - \eta^2 - \xi^2) f(\mathrm{Fo})$ inside a rectangular bar in the space $\{\psi_k(\xi, \eta) = (1-\xi^2)(1-\eta^2)\xi^{2(k-1)}\eta^{2(k-1)}\}$ along the first coordinate axis we find

$$\bar{a}_1(p) = \frac{q_v h^2}{\lambda} \frac{2.5\bar{f}(p)}{p + 2.5(1+\beta^2)}, \tag{46}$$

whence and using the representation (45) the solutions in the first approximation for the two forms $f(\mathrm{Fo}) = 1$ and $f(\mathrm{Fo}) = 1 - \exp(-\mathrm{Pd\ Fo})$ are reduced to the expressions

$$T(\xi, \eta, \mathrm{Fo}) = T_0 + \frac{q_v h^2}{\lambda(1+\beta^2)}(1-\xi^2)(1-\eta^2) \times$$

$$\times \left\{1 - \exp[-A(\beta)\,\mathrm{Fo}]\right\},\quad A(\beta) = 2.5(1+\beta^2), \tag{47}$$

$$T(\xi, \eta, \mathrm{Fo}, \mathrm{Pd}) = T_0 + \frac{q_v h^2}{\lambda(1+\beta^2)}(1-\xi^2)(1-\eta^2) \times$$

$$\times \left\{1 - \frac{\mathrm{Pd}\exp[-A(\beta)\,\mathrm{Fo}] - A(\beta)\exp(-\mathrm{Pd\ Fo})}{\mathrm{Pd} - A(\beta)}\right\}. \tag{48}$$

Only inside a square bar ($\beta = 1$) do these solutions after the transient regime coincide with the exact solution of the stationary problem.

For the internal heat source of induction heating (44) and constant boundary conditions of the first kind, the solution of the best approximation will be

$$T(\xi, \eta, \mathrm{Fo}) = T_0 + \frac{5q_v h^2}{64\lambda}(1-\xi^4)(1-\eta^4)\left\{1 - \exp(-6.429\,\mathrm{Fo})\right\}, \tag{49}$$

in which the generalized isothermal lines are similar at any instant to the exact isothermal lines of the stationary problem.

If, in a square bar, we set $q_v(\xi, \eta, \mathrm{Fo}) = q_v \cos\mu_i\xi \cos\mu_j\eta f(\mathrm{Fo})$, where μ_i and μ_j are any fixed roots of the equation $\cot\mu = \mu/\mathrm{Bi}$ realization of the method in the space $\{\psi_{kn}(\xi, \eta) = \cos\mu_k\xi \cos\mu_n\eta\}$ causes the internal source to seek its eigensolution in the form

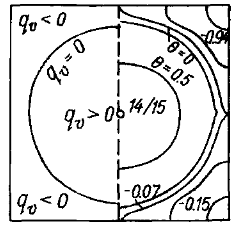

Fig. 2. Isothermal surfaces in a square bar and the interface of sign inversion in the heat source.

$$T(\xi, \eta, \mathrm{Fo}) = T_0 + \frac{q_v h^2}{\lambda} \cos \mu_i \xi \cos \mu_j \eta \times$$

$$\times \left\{ \int_0^{\mathrm{Fo}} f(\tau) \exp\left[-(\mu_i^2 + \mu_j^2)(\mathrm{Fo} - \tau)\right] d\tau \right\}. \tag{50}$$

Let us consider another distribution $q_v(\xi, \eta, \mathrm{Fo}) = q_v(2 - 3\xi^2 - 3\eta^2)f(\mathrm{Fo})$ where $q_v(\xi, \eta, \mathrm{Fo}) > 0$ inside the circle $\xi^2 + \eta^2 < 2/3$ and $q_v(\xi, \eta, \mathrm{Fo}) < 0$ in the remaining part of the square $D\{-1 \le \xi \le 1, -1 \le \eta \le 1\}$.

The nonstationary temperature field inside a bar fuel element of square shape with adiabatic walls and an alternating stationary distribution ($f(\mathrm{Fo}) = 1$) is found in the best approximation by the formula

$$T(\xi, \eta, \mathrm{Fo}) = T_0 + \frac{q_v R^2}{4\lambda}\left(\xi^4 - 2\xi^2 + \eta^4 - 2\eta^2 + \frac{14}{15}\right)\left\{1 - \exp(-10\,\mathrm{Fo})\right\}, \tag{51}$$

in which the exact temperature distribution is established after the transient regime ($\exp(-10\mathrm{Fo}) \approx 0$). The isotherms $\theta(\xi, \eta) \approx 4\lambda(T - T_0)/q_v h^2[1 - \exp(-10\mathrm{Fo})]$ and the boundary of the zones of sign inversion of $q_v(\xi, \eta, \mathrm{Fo})$ are given in Fig. 2. We note that after redistribution of the uniform initial temperature T_0 a zero isothermal line ($\theta = 0$, $T = T_0$) is established in the zone where heat is absorbed. Whereas in induction heating and heat insulation of the walls from the external medium the temperature of the body increases linearly [1] with time, here enthalpy increase is absent.

Determination of the temperature inside a bar of isosceles triangular cross section with the heat source $q_v(\xi, \eta, \mathrm{Fo}) = q_v f(\mathrm{Fo})$ with a constant temperature on the sides equal to T_0 as the principal response along the first coordinate axis for $\psi_1(\xi, \eta) = \omega(\xi, \eta) = (\xi^2 - \eta^2)(1 - \xi)$ leads to the formula

$$\bar{a}_1(p) = \frac{10.5 q_v h^2}{\lambda} \frac{\bar{f}(p)}{p + A(\beta)}, \quad A(\beta) = 7(\beta^2 + 3), \tag{52}$$

whence the temperature with constant sources ($\bar{f}(p) = 1/p$) is found in the form

$$T(\xi, \eta, \mathrm{Fo}, \beta) = T_0 + \frac{3 q_v h^2}{2\lambda(\beta^2 + 3)}(\xi^2 - \eta^2)(1 - \xi)\left\{1 - \exp\left[-A(\beta)\,\mathrm{Fo}\right]\right\}. \tag{53}$$

In this solution, only for an equilateral triangular cross section will the isothermal lines at any instant, which are similar to a closed composite boundary function $\omega(\xi, \eta) > 0$, coincide with the isothermal lines of the stationary temperature of the exact solution of the problem formulated.

Inside an isosceles triangle with a right apex angle we will consider ($\beta = 1$) a heat source whose stationary distribution increases linearly from zero to $q_v = \mathrm{const}$ along the height of the triangle, i.e., $q_v(\xi, \eta, \mathrm{Fo}) = q_v \xi f(\mathrm{Fo})$. Then instead of formula (52) we will have

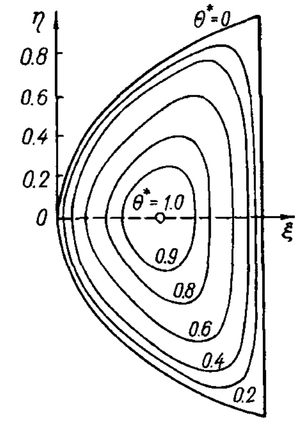

Fig. 3. Generalized isothermal surfaces that are similar to the composite boundary function of a parabolic segment and are caused by a special distribution of the heat sources.

$$\bar{a}_1(p) = \frac{q_v h^2}{4\lambda} \frac{p_1 \bar{f}(p)}{p + p_1}, \quad p_1 = 28, \tag{54}$$

which will enable us to find the temperature as in all other cases for any prescribed control function $f(\mathrm{Fo})$. For example, for $f(\mathrm{Fo}) = 1$

$$T(\xi, \eta, \mathrm{Fo}) = T_0 + \frac{q_v h^2}{4\lambda} (\xi^2 - \eta^2)(1 - \xi)\{1 - \exp(-28\,\mathrm{Fo})\}, \tag{55}$$

where now the composite boundary function $\omega(\xi, \eta)$ will coincide with the isothermal lines of the exact solution of the stationary temperature of the steady-state regime. In the general case, it is required that the form of the distribution of the heat source $q_v(\xi, \eta, \mathrm{Fo}) = q_v \varphi_0(\xi, \eta) f(\mathrm{Fo})$ be found for which the composite function of the closed profile of the bar cross section $\omega(\xi, \eta)$ will describe generalized isothermal lines, i.e., will coincide with $\theta(\xi, \eta)$ in formula (42). The function $\varphi_0(\xi, \eta)$ is determined accurate to a constant factor and is equal to the expression $\frac{\partial^2 \omega}{\partial^2 \xi^2} + \beta^2 \frac{\partial^2 \omega}{\partial \eta^2}$. For example, for an arbitrary isosceles triangle this expression is equal to $-2[(\beta^2 - 1) + (3 - \beta^2)\xi]$, whence for an equilateral triangle $\beta^2 = 3$ and $\varphi_0(\xi, \eta) = 1 = \mathrm{const}$, and for a right triangle $\beta^2 = 1$ and $\varphi_0 = \xi$. For the parabolic segment $D\{x \geq \frac{h}{b^2} y,\ 0 \leq x \leq h\}$ we found the function $\omega(\xi, \eta) = (\xi - \eta^2)(1 - \xi)$, $\xi = x/h$, $\eta = y/b$. Therefore $\frac{\partial^2 \omega}{\partial \xi^2} + \beta^2 \frac{\partial^2 \omega}{\partial \eta^2} = -2[1 + \beta^2(1 - \xi)]$ and when a constant temperature is maintained on the fuel-element walls the internal source $q_v[1 + \beta^2(1 - \xi)]f(\mathrm{Fo})$ produces a thermal state for which the isothermal lines at any instant are quasisimilar to the composite equation of the cross-sectional profile. The temperature inside such a bar with the parameters $h = b$ ($\beta = 1$) for the source $q_v(\xi, \eta, \mathrm{Fo}) = q_0(2 - \xi)f(\mathrm{Fo})$ is found as

$$T(\xi, \eta, \mathrm{Fo}) = T_0 + \frac{q_v h^2}{2\lambda} (\xi - \eta^2)(1 - \xi) A(\beta) \times$$

$$\times \int_0^{\text{Fo}} f(\tau) \exp[-A(\beta)(\text{Fo}-\tau)]\, d\tau\,, \quad A(1) = 17.873\,. \tag{56}$$

According to formula (42),

$$\theta(\xi, \eta) = 0.5\,(\xi - \eta^2)(1-\xi)\,, \quad Q(\text{Fo}) = A(1) \int_0^{\text{Fo}} f(\tau) \exp[-A(1)(\text{Fo}-\tau)]\, d\tau$$

and the largest value is $\theta(0.5, 0) = 0.125$. Closed generalized isothermal lines for $\theta^* = 8\theta$ that are calculated by the formula $\eta = \pm\left(\xi - \dfrac{\theta^*}{4(1-\xi)}\right)^{1/2}$ are given in Fig. 3.

To compare the rate of stabilization of the solution with the stabilization of the temperatures in round and triangular bars, we refer the Fo number to the equivalent radius $\tilde{R} = 2S/\mathscr{P}$, where $S = \frac{4}{3}h^2$; $\mathscr{P} = 4d + 2h$; d is the length of four equal chords inscribed in a parabolic arc. Then $\tilde{R} = 0.505h$ and the first eigenvalue is $p_1 = 4.559$, while for an equilateral triangle and a circle, $\mu_1^2 = 4.525$ and $\mu_1^2 = 5.783$, respectively.

Investigation of problems of nonstationary heat conduction in three-dimensional axisymmetric bodies of revolution ($\xi = \xi$, $\eta = \sqrt{\eta^2 + \zeta^2}$, $\zeta = z/b$) leads to solutions of intermediate boundary-value problems for the equation

$$\frac{\partial^2 \bar{T}}{\partial \xi^2} + \beta^2 \left(\frac{\partial^2 \bar{T}}{\partial \eta^2} + \frac{\partial^2 \bar{T}}{\partial \zeta^2}\right) - \left[p\bar{T}(\xi, \eta, \zeta, p) - T_0\right] + \frac{q_v h^2}{\lambda} \varphi_0(\xi, \eta, \zeta) \bar{f}(p) = 0. \tag{57}$$

Thus, whereas in classical methods of mathematical physics the heat-conduction equation is transformed, depending on the shape of the body, to cylindrical, spherical, and other curvilinear coordinates, in the computational algorithm proposed the equations of the processes of nonstationary heat conduction for bodies of any geometric shape are solved in a rectangular coordinate system x, y, z, and the variety of the representation of the solutions in different alternative Riemannian spaces is achieved by a wide possibility for selecting the system of basis coordinates, depending on the geometry of the body and the internal and external conditions of thermal loadings, which made it possible to express the temperature fields in the best approximations by simple functional dependences.

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